

**Barem clasa a VII-a**  
**(OLM 2015-etapa locală)**

**Of. 10 p**

**Subiectul I. (20 puncte)**

$$a = \frac{1}{2} \left( 1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{4} + \frac{1}{3} - \frac{1}{5} - \frac{1}{4} + \frac{1}{6} + \dots - \frac{1}{2n-2} + \frac{1}{2n} + \frac{1}{2n-1} - \frac{1}{2n+1} \right) = \frac{1}{2} \left( 1 - \frac{1}{2} + \frac{1}{2n} - \frac{1}{2n+1} \right) = \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2n(2n+1)} \right) =$$

$$\frac{1}{4} \cdot \frac{n(2n+1)+1}{n(2n+1)} = \frac{1}{4} \cdot \frac{2n^2+n+1}{2n^2+n} \quad (10 \text{ p})$$

$$\frac{1}{4} < \frac{1}{4} \cdot \frac{2n^2+n+1}{2n^2+n} \Leftrightarrow 2n^2 + n < 2n^2 + n + 1 \quad "A" \quad (5 \text{ p})$$

$$\frac{1}{4} \cdot \frac{2n^2 + n + 1}{2n^2 + n} \leq \frac{1}{3} \Leftrightarrow 6n^2 + 3n + 3 \leq 8n^2 + 4n \Leftrightarrow 2n^2 + n \geq 3 \Leftrightarrow n(2n + 1) \geq 3 \quad "A" \quad (5 \text{ p})$$

**Subiectul II. (30 puncte )**

Desen corect

(5 p)

a)  $[AE \text{ bisectoare}] \Rightarrow \frac{BE}{EC} = \frac{AB}{AC} \quad (5 \text{ p})$

$$[AG \text{ bisectoare}] \Rightarrow \frac{BG}{GF} = \frac{AB}{AF} \Rightarrow \frac{BG}{GF} = \frac{AB}{\frac{AC}{2}} \Rightarrow \frac{BG}{GF} = \frac{2AB}{AC} \quad ; \quad \frac{BG}{GF} \cdot \frac{CE}{BE} = \frac{2AB}{AC} \cdot \frac{AC}{AB} = 2 \Rightarrow a = 1. \quad (5 \text{ p})$$

b)  $A_{\triangle BEG} = A_{\triangle BEA} - A_{\triangle BGA}$

$$A_{\triangle AGF} = A_{\triangle BFA} - A_{\triangle BGA} \Rightarrow A_{\triangle BEA} = A_{\triangle AFB} \Rightarrow \frac{BA \cdot EM}{2} = \frac{BA \cdot FN}{2} \Rightarrow EM = FN \quad (10 \text{ p})$$

$EM \perp AB, FN \perp AB \Rightarrow EM \parallel FN \Rightarrow EF \parallel AB, F$  mijlocul segmentului  $AC \Rightarrow E$  mijlocul segmentului  $BC \Rightarrow G$  centrul de greutate al  $\triangle ABC$ . (5 p)

**Subiectul III. (20 puncte )**

Desen corect

(5 p)

$$[CE, [BE \text{ sunt bisectoare}] \Rightarrow m(\angle ECB) + m(\angle EBC) = 180^\circ : 2 = 90^\circ \Rightarrow m(\angle CEB) = 90^\circ \quad (5 \text{ p})$$

$$F \text{ este mijlocul lui } (BC) \Rightarrow FE = \frac{BC}{2} = 6 = FB = FC \Rightarrow \triangle EFB \text{ isoscel}$$

$$\Rightarrow \left. \begin{array}{l} \angle FEB \equiv \angle FBE \\ \text{dar } \angle ABE \equiv \angle FBE \end{array} \right\} \Rightarrow \angle FEB \equiv \angle ABE \Rightarrow EF \parallel AB \quad (5 \text{ p})$$

Deci  $EF$  este linie mijlocie  $AB + CD = 12$ , dar  $AB - CD = 8$ , deci  $AB = 10$

$$\text{În } \triangle ABC, AC < AB + BC = 22 \text{ cm} \quad (5 \text{ p})$$

**Subiectul IV. (20 puncte )**

Dacă  $n$  este impar,  $n = 2k + 1$ , atunci  $35^{2k+1} = 35^{2k} \cdot 35 = 35^{2k} (36 - 1) = (35^k \cdot 6)^2 - (35^k)^2 \quad (10 \text{ p})$

Dacă  $n$  este par,  $n = 2k$ , atunci  $35^{2k} = 35^{2k-2} \cdot 35^2 = 35^{2k-2} \cdot 1225 = 35^{2k-2} \cdot (37^2 - 12^2) = (35^{k-1} \cdot 37)^2 - (35^{k-1} \cdot 12)^2 \quad (10 \text{ p})$